A Bilinear Equalizer for Massive MIMO Systems

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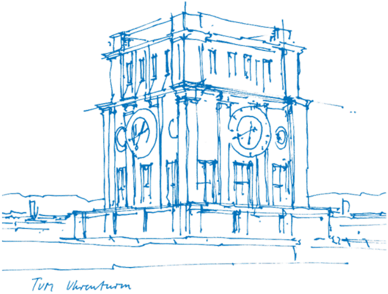

[1] see: arXiv:1707.09940

1. Bilinear Equalization

CSI Acquisition

transmission of orthogonal pilots

estimation of channel by correlation:

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MMSE estimate:

$$\hat{\boldsymbol{h}}_k = \boldsymbol{C}_k \boldsymbol{Q}_k^{-1} \boldsymbol{\varphi}_k$$

with $\boldsymbol{C}_k = \mathrm{E}[\boldsymbol{h}_k \boldsymbol{h}_k^{\mathrm{H}}]$ and $\boldsymbol{Q}_k = \boldsymbol{C}_k + \frac{1}{\rho_{\text{tr}}} \mathbf{I}$

Pilot Contamination

transmission of orthogonal pilots
but more users than pilots
reuse of pilots

interference in estimation of channel by correlation:

$$\boldsymbol{\varphi}_k = \boldsymbol{h}_k + \sum_{n \in \mathcal{I}_k} \boldsymbol{h}_n + \frac{1}{\sqrt{\rho_{\text{tr}}}} \boldsymbol{w}_k$$

with $\mathcal{I}_k$: set of users with the same pilot sequence as user k

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Data Transmission and Equalization

received signal in the uplink

$$\boldsymbol{y} = \sum_k \sqrt{p}_k \boldsymbol{h}_k s_k + \boldsymbol{v}$$

with $s_k \sim \mathcal{N}_{\mathbb{C}}(0, 1)$ and $\boldsymbol{v} \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \mathbf{I})$

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data estimate for user k:

$$\begin{aligned} \hat{s}_k &= \boldsymbol{g}_k^{\mathrm{H}} \boldsymbol{y} \\ &= \sqrt{p_k} \boldsymbol{g}_k^{\mathrm{H}} \boldsymbol{h}_k s_k + \sum_{j \neq k} \sqrt{p_j} \boldsymbol{g}_k^{\mathrm{H}} \boldsymbol{h}_j s_j + \boldsymbol{g}_k^{\mathrm{H}} \boldsymbol{v} \end{aligned}$$

Equalization with Matched Filter

estimate based on matched filter (good equalization for massive MIMO)

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or MMSE channel estimate

$$\hat{s}_k = \boldsymbol{\varphi}_k^{\mathrm{H}} \boldsymbol{Q}_k^{-1} \boldsymbol{C}_k \boldsymbol{y}$$

Bilinear Equalization

depending on the employed channel estimator

$$\boldsymbol{g}_k^{\mathrm{H}} = \boldsymbol{\varphi}_k^{\mathrm{H}} \boldsymbol{A}_k$$

with deterministic $\boldsymbol{A}_k$ (independent of actual CSI)

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therefore, generalized matched filter

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estimate

$$\hat{s}_k = \boldsymbol{\varphi}_k^{\mathrm{H}} \boldsymbol{A}_k \boldsymbol{y}$$

is linear in $\boldsymbol{y}$ and also linear in $\boldsymbol{\varphi}_k$

thus, bilinear equalization

2. Asymptotic Properties of SINR Maximization

SINR Maximization

use Medard's bound

$$I(s_k; \hat{s}_k) \geq \log_2(1 + \gamma_k^{\mathsf{ul}})$$

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optimal SINR can be written as

$$\gamma_k^\star = M p_k \frac{\boldsymbol{e}_k^{\mathrm{T}} \boldsymbol{\Gamma} (\frac{1}{M}\boldsymbol{P}^{-1} + \boldsymbol{\Gamma})^{-1} \boldsymbol{e}_k}{\boldsymbol{e}_k^{\mathrm{T}} (\frac{1}{M}\boldsymbol{P}^{-1} + \boldsymbol{\Gamma})^{-1} \boldsymbol{e}_k}$$

with $[\boldsymbol{\Gamma}]_{nk} = \frac{1}{M}\, \mathrm{tr}\left(\boldsymbol{C}_n \boldsymbol{Z}^{-1} \boldsymbol{C}_k \boldsymbol{Q}_k^{-1}\right)$ and $\boldsymbol{Z} = \mathbf{I} + \sum_n p_n \boldsymbol{C}_n$

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for large M,

$$\gamma_k^\star \to M \frac{p_k}{\boldsymbol{e}_k^{\mathrm{T}} \boldsymbol{\Gamma}^{-1} \boldsymbol{e}_k} = \gamma_k^{\text{asy}}$$

Asymptotic Properties

for linearly independent covariance matrices $\boldsymbol{C}_k$, $k \in \Omega_p$

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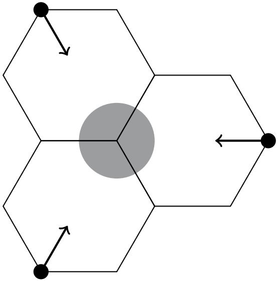
has been demonstrated for the SINR depending on CSI in [Björnson, Hoydis, Sanguinetti, arXiv:170500538, May 2017]

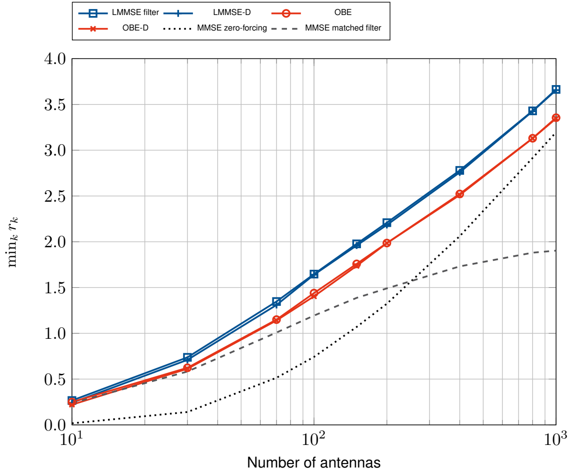
also holds for application of approximate covariance matrices $\boldsymbol{C}_k$

Simulation Scenario



5 users in each cell, only 5 pilots

Simulation w.r.t. Number of BS Antennas



Simulation w.r.t. SNR